

Secretly Broadcasting Five Qubit Entangled state among three parties from W- type states

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Abstract

In this work we investigate the problem of secretly broadcasting five qubit entangled state between three different partners We implement the protocol described in ref [16] on three particle W-state shared by three distant partners Alice,Bob and Charlie. The problem is interesting in the sense it is the first attempt to broadcast five qubit entangled state between three parties.

1 Introduction

Linearity of quantum theory doesn't allow us to amplify and delete an arbitrary quantum state [1][6]. Although nature prevents us from amplifying an unknown quantum state but nevertheless one can always design a quantum cloning machine that duplicates an unknown quantum state with a fidelity less than unity [1,2,3,4,5].

However, the authors in [7] if the states are linearly independent, they do can be cloned by a unitary-reduction process. In the past years,much progress has been made in designing

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quantum cloning machine. Buzek-Hillery took the first step towards the construction of approximate quantum cloning machine [2]. This machine is known as universal quantum cloning machine (UQCM) as the quality of the copies produced by their machine remain same for all input state. Later Gisin -Massar showed the machine to be optimal [3]. After that the different sets of quantum cloning machines like the set of universal quantum cloning machines, the set of state dependent quantum cloning machines (i.e. the quality of the copies depend on the input state) and the probabilistic quantum cloning machines were proposed. Entanglement[8] plays a crucial role in computational and communicational purposes and is used as a valuable resource in quantum information processing, quantum entanglement quantum cryptography [9,10], quantum super dense coding [11] and quantum teleportation [12]. An interesting feature of quantum information processing is that information can be encoded in non-local correlations between two separated particles. Pure quantum entanglement, is more valuable. Therefore, it becomes interesting to extract pure quantum entanglement from a partially entangled state [10]. In other words, it is possible to compress locally an amount of quantum information. Now one can ask an question : whether the opposite is true or not i.e. can quantum correlations be decompressed? This question was tackled by several researchers using the concept of Broadcasting of quantum inseparability. Broadcasting is nothing but a local copying of non-local quantum correlations. That is the entanglement originally shared by a single pair is transferred into two less entangled pairs using only local operations.

Suppose two distant parties A and B share two qubit-entangled state

$$|\psi\rangle_{AB} = \alpha|00\rangle_{AB} + \beta|11\rangle_{AB} \quad (1)$$

The first qubit belongs to A and the second belongs to B. Each of the two parties now perform local copier on their own qubit and then the input entangled state has been broadcast if for some values of the probability α^2

- (1) non-local output states are inseparable, and
- (2) local output states are separable.

V.Buzek et.al. showed that the decompression of initial quantum entanglement is theoretically possible, i.e. if we start with an entangled pair, then two less entangled pairs

can be obtained by local operations. That means inseparability of quantum states can be partially broadcasted (cloned) with the help of local operation. They used optimal universal quantum cloners for local copying of the subsystems and showed that the non-local outputs are inseparable if α^2 lies in the interval $(\frac{1}{2} - \frac{\sqrt{39}}{16}, \frac{1}{2} + \frac{\sqrt{39}}{16})$.

Further S.Bandyopadhyay et.al. [13] showed that only those universal quantum cloners whose fidelity is greater than $\frac{1}{2}(1 + \sqrt{\frac{1}{3}})$ are suitable because only then the non-local output states becomes inseparable for some values of the input parameter α and also proved that an entanglement is optimally broadcast only when optimal quantum cloners are used for the purpose of local copying. They also showed that broadcasting of entanglement into more than two entangled pairs is not possible using only local operations. I.Ghiu investigated the broadcasting of entanglement by using local $1 \rightarrow 2$ optimal universal asymmetric Pauli machines and showed that the inseparability is optimally broadcast when symmetric cloners are applied [14]. Few years back we study the problem of broadcasting of entanglement using state dependent quantum cloning machine as a local copier. We show that the length of the interval for probability-amplitude-squared for broadcasting of entanglement using state dependent cloner can be made larger than the length of the interval for probability-amplitude-squared for broadcasting entanglement using state independent cloner. In that work we showed that there exists local state dependent cloner which gives better quality copy (in terms of average fidelity) of an entangled pair than the local universal cloner [15].

In recent past Adhikari et.al in their paper [16] showed that secretly broadcasting of three-qubit entangled state between two distant partners with universal quantum cloning machine is possible. They generalized the result to generate secret entanglement among three parties. Recently Adhikari et.al in ref [17] proposed a scheme for broadcasting of continuous variable entanglement. In a recent work we have presented a scheme of broadcasting W state secretly between three distant partners [18].

Motivated by ref [16] we investigate the problem of secretly broadcasting Five qubit entangled state between three distant Alice, Bob and Charlie. In this work we consider a W type of state

$$|X\rangle_{123} = \alpha|001\rangle_{123} + \beta|010\rangle_{123} + \gamma|100\rangle_{123} \quad (2)$$

The three parties then apply optimal universal quantum cloning machine on their respective qubits to produce six qubit state. One of the party (say, Alice) then performs measurement on her quantum cloning machine state vectors. Bob also performs measurement on his quantum cloning machine state vectors. Charlie also does measurement on his cloning state vectors. Each party inform others about their measurement result using Goldenberg and Vaidmans quantum cryptographic scheme [20] based on orthogonal state. Since the measurement results are interchanged secretly so Alice, Bob and Charlie share secretly six qubit state. They again apply the cloning machine on one of their respective qubits and generate nine qubit state. Now once again each of them do measurement on their machine state vectors and secretly inform each other about measurement outcomes. Interestingly, we find a five qubit entangled state between the original qubit of Alice and two cloned copies from each of Bob's and Charlie's subsystem. We observe that the local output states are also separable. Thus we are able to broadcast a five qubit entangled state among three distant partners. Since three parties perform measurements twice on their machine state vectors and communicate with each other secretly, so the final five qubit entangled state shared by them can be used as a secret quantum channel between these parties. Any fourth party has to obtain the information about the measurement results in order to know about the five qubit entangled state.

In broadcasting of inseparability, we generally use Peres-Horodecki criteria [21,22] to show the inseparability of non-local outputs and separability of local outputs.

Peres-Horodecki Theorem : The necessary and sufficient condition for the state ρ of two spins $\frac{1}{2}$ to be inseparable is that at least one of the eigen values of the partially transposed operator defined as $\rho_{m\mu,n\nu}^T = \rho_{m\mu,n\nu}$, is negative. This is equivalent to the condition that at least one of the two determinants

$$W_3 = \begin{vmatrix} \rho_{00,00} & \rho_{01,00} & \rho_{00,10} \\ \rho_{00,01} & \rho_{01,01} & \rho_{00,11} \\ \rho_{10,00} & \rho_{11,00} & \rho_{10,10} \end{vmatrix} \text{ and } W_4 = \begin{vmatrix} \rho_{00,00} & \rho_{01,00} & \rho_{00,10} & \rho_{01,10} \\ \rho_{00,01} & \rho_{01,01} & \rho_{00,11} & \rho_{01,11} \\ \rho_{10,00} & \rho_{11,00} & \rho_{10,10} & \rho_{11,10} \\ \rho_{10,01} & \rho_{11,01} & \rho_{10,11} & \rho_{11,11} \end{vmatrix}$$

is negative.

The protocol is interesting in the sense that, this is a first attempt to broadcast five qubit entangled state from a three qubit W-state using B-H local quantum copying machine.

2 Local Copying of W-type state and Broadcasting

Let three parties Alice, Bob and Charlie share a W-type state of the form

$$|X\rangle_{123} = \alpha|001\rangle_{123} + \beta|010\rangle_{123} + \gamma|100\rangle_{123} \quad (3)$$

where without any loss of generality, we have assumed that α, β, γ are all real with $\alpha^2 + \beta^2 + \gamma^2 = 1$. The qubits 1,2,3 are in possession with Alice, Bob and Charlie respectively.

The B-H cloning transformation is given by,

$$\begin{aligned} |0\rangle &\longrightarrow \sqrt{\frac{2}{3}}|00\rangle|\uparrow\rangle + \frac{1}{\sqrt{6}}(|01\rangle + |10\rangle)|\downarrow\rangle \\ |1\rangle &\longrightarrow \sqrt{\frac{2}{3}}|11\rangle|\downarrow\rangle + \frac{1}{\sqrt{6}}(|01\rangle + |10\rangle)|\uparrow\rangle \end{aligned} \quad (4)$$

where $\{|\uparrow\rangle, |\downarrow\rangle\}$ are orthogonal quantum cloning machine state vectors.

Now Alice, Bob and Charlie apply the cloning machine defined by equation (4) on their respective qubits and hence obtain six qubit state given by,

$$\begin{aligned} |X_1\rangle_{142536} = & \alpha \left[\sqrt{\frac{2}{3}}|00\rangle_{14}|\uparrow\rangle^A + \frac{1}{\sqrt{6}}(|01\rangle_{14} + |10\rangle_{14})|\downarrow\rangle^A \right] \otimes \\ & \left[\sqrt{\frac{2}{3}}|00\rangle_{25}|\uparrow\rangle^B + \frac{1}{\sqrt{6}}(|01\rangle_{25} + |10\rangle_{25})|\downarrow\rangle^B \right] \otimes \\ & \left[\sqrt{\frac{2}{3}}|11\rangle_{36}|\downarrow\rangle^C + \frac{1}{\sqrt{6}}(|01\rangle_{36} + |10\rangle_{36})|\uparrow\rangle^C \right] \\ & + \beta \left[\sqrt{\frac{2}{3}}|00\rangle_{14}|\uparrow\rangle^A + \frac{1}{\sqrt{6}}(|01\rangle_{14} + |10\rangle_{14})|\downarrow\rangle^A \right] \otimes \\ & \left[\sqrt{\frac{2}{3}}|11\rangle_{25}|\downarrow\rangle^B + \frac{1}{\sqrt{6}}(|01\rangle_{25} + |10\rangle_{25})|\uparrow\rangle^B \right] \otimes \\ & \left[\sqrt{\frac{2}{3}}|00\rangle_{36}|\uparrow\rangle^C + \frac{1}{\sqrt{6}}(|01\rangle_{36} + |10\rangle_{36})|\downarrow\rangle^C \right] \end{aligned}$$

$$\begin{aligned}
& +\gamma[\sqrt{\frac{2}{3}}|11\rangle_{14}|\downarrow\rangle^A + \frac{1}{\sqrt{6}}(|01\rangle_{14} + |10\rangle_{14})|\uparrow\rangle^A] \otimes \\
& [\sqrt{\frac{2}{3}}|00\rangle_{25}|\uparrow\rangle^B + \frac{1}{\sqrt{6}}(|01\rangle_{25} + |10\rangle_{25})|\downarrow\rangle^B] \otimes \\
& [\sqrt{\frac{2}{3}}|00\rangle_{36}|\uparrow\rangle^C + \frac{1}{\sqrt{6}}(|01\rangle_{36} + |10\rangle_{36})|\downarrow\rangle^C]
\end{aligned} \tag{5}$$

where qubits with subscripts '4', '5', '6' are cloned copies of the qubits with subscripts '1', '2', '3' respectively.

Now if Alice, Bob, Charlie carry out measurements on their machine state vectors $\{|\uparrow\rangle^A, |\downarrow\rangle^A\}, \{|\uparrow\rangle^B, |\downarrow\rangle^B\}, \{|\uparrow\rangle^C, |\downarrow\rangle^C\}$ and exchange the measurement results with each other secretly with the help of Goldenberg and Vaidmans quantum cryptographic scheme based on orthogonal state [20].

The tensor products of machine state vectors after the measurement is given by the following table.

TABLE 1:

Serial Number	Measurement Results
1	$ \uparrow\rangle^A \uparrow\rangle^B \uparrow\rangle^C$
2	$ \uparrow\rangle^A \uparrow\rangle^B \downarrow\rangle^C$
3	$ \uparrow\rangle^A \downarrow\rangle^B \downarrow\rangle^C$
4	$ \uparrow\rangle^A \downarrow\rangle^B \uparrow\rangle^C$
5	$ \downarrow\rangle^A \uparrow\rangle^B \uparrow\rangle^C$
6	$ \downarrow\rangle^A \uparrow\rangle^B \downarrow\rangle^C$
7	$ \downarrow\rangle^A \downarrow\rangle^B \uparrow\rangle^C$
8	$ \downarrow\rangle^A \downarrow\rangle^B \downarrow\rangle^C$

Now if we assume that the measurement outcome is $|\uparrow\rangle^A|\uparrow\rangle^B|\uparrow\rangle^C$, then the six qubit post measurement state shared by Alice, Bob and Charlie is given by,

$$|X_2\rangle_{142536} = \frac{1}{\sqrt{N}}\{\alpha(|000001\rangle + |000010\rangle) + \beta(|000100\rangle + |001000\rangle) + \gamma(|010000\rangle + |100000\rangle)\} \tag{6}$$

where $N = \sqrt{2\alpha^2 + 2\beta^2 + 2\gamma^2}$ is the normalization constant.

Now once again three distant partners apply local cloning operations given by equaton

(4) on their qubits '4','5','6' respectively and performs measurement in the machine state vectors. They also exchange the measurement results once again (table1) using the same cryptographic scheme.

Again if we assume that the measurement outcome is $|\uparrow\rangle^A|\uparrow\rangle^B|\uparrow\rangle^C$, then the nine qubit state shared by Alice, Bob and Charlie is given by,

$$\begin{aligned} |X_3\rangle_{147258369} = \frac{1}{\sqrt{N_1}} \{ & \alpha(x|000000001\rangle + x|000000010\rangle + y|000000100\rangle) \\ & + \beta(x|000001000\rangle + x|000010000\rangle + y|000100000\rangle) \\ & + \gamma(x|001000000\rangle + x|010000000\rangle + y|100000000\rangle) \} \end{aligned} \quad (7)$$

where N_1 is the normalization constant,

and $x = \frac{\sqrt{2}}{3\sqrt{3}}, y = \frac{2\sqrt{2}}{3\sqrt{3}}$.

Now if Alice applies σ_z operator on her cloned qubits '4' and '7' and Bob, Charlie apply σ_y on their original qubits 2 and 3 respectively, then the nine qubit entangled state takes the form,

$$\begin{aligned} |\bar{X}_3\rangle_{147258369} = \frac{1}{\sqrt{N_1}} \{ & \alpha(-x|011100101\rangle - x|011100110\rangle + y|011100000\rangle) \\ & + \beta(-x|011101100\rangle - x|011110100\rangle + y|011000100\rangle) \\ & + \gamma(-x|010100100\rangle - x|001100100\rangle + y|111100100\rangle) \} \end{aligned} \quad (8)$$

Interestingly we find a five qubit state shared by three parties given by the density matrix,

$$\begin{aligned} \rho_{15869} = \frac{1}{N_1} \{ & x^2\alpha^2|00001\rangle\langle 00001| + x^2\alpha^2|00001\rangle\langle 00010| + x^2\alpha\beta|00001\rangle\langle 00100| \\ & + x^2\alpha\beta|00001\rangle\langle 01000| + xy\alpha\gamma|00001\rangle\langle 10000| + x^2\alpha^2|00010\rangle\langle 00010| + x^2\alpha^2|00010\rangle\langle 00001| \\ & + x^2\alpha\beta|00010\rangle\langle 00100| + x^2\alpha\beta|00010\rangle\langle 01000| + xy\alpha\gamma|00010\rangle\langle 10000| \\ & + y^2\alpha^2|00000\rangle\langle 00000| + x^2\beta^2|00100\rangle\langle 00100| + x^2\alpha\beta|00100\rangle\langle 00001| \\ & + x^2\alpha\beta|00100\rangle\langle 00010| + x^2\beta^2|00100\rangle\langle 01000| + xy\beta\gamma|00100\rangle\langle 10000| \\ & + x^2\beta^2|01000\rangle\langle 01000| + x^2\beta^2|01000\rangle\langle 00100| + x^2\alpha\beta|01000\rangle\langle 00001| + x^2\alpha\beta|01000\rangle\langle 00010| \\ & + xy\beta\gamma|01000\rangle\langle 10000| + y^2\beta^2|00000\rangle\langle 00000| + x^2\gamma^2|00000\rangle\langle 00000| \\ & + x^2\gamma^2|00000\rangle\langle 00000| + y^2\gamma^2|10000\rangle\langle 10000| + xy\alpha\gamma|10000\rangle\langle 00001| \\ & + xy\alpha\gamma|10000\rangle\langle 00010| + xy\beta\gamma|10000\rangle\langle 00100| + xy\beta\gamma|10000\rangle\langle 01000| \} \end{aligned} \quad (9)$$

In order to show that the above five qubit states are entangled , we have to show that each of the two qubit density matrices $\rho_{15}, \rho_{16}, \rho_{58}, \rho_{69}, \rho_{86}$, is entangled. Simultaneously we must show that the local output states $\rho_{17}, \rho_{14}, \rho_{25}, \rho_{28}, \rho_{36}, \rho_{39}$ are separable.

Now, the non local output states are given as,

$$\begin{aligned} \rho_{15} = \frac{1}{N_1} \{ & x^2 \alpha^2 |00\rangle \langle 00| + x^2 \alpha^2 |00\rangle \langle 00| + y^2 \alpha^2 |00\rangle \langle 00| \\ & + x^2 \beta^2 |00\rangle \langle 00| + x^2 \beta^2 |01\rangle \langle 01| + xy \beta \gamma |01\rangle \langle 10| \\ & + y^2 \beta^2 |00\rangle \langle 00| + x^2 \gamma^2 |00\rangle \langle 00| + x^2 \gamma^2 |00\rangle \langle 00| \\ & + y^2 \gamma^2 |10\rangle \langle 10| + xy \beta \gamma |10\rangle \langle 01| \} \end{aligned} \quad (10)$$

$$\begin{aligned} \rho_{58} = \frac{1}{N_1} \{ & x^2 \alpha^2 |00\rangle \langle 00| + x^2 \alpha^2 |00\rangle \langle 00| + x^2 \alpha^2 |00\rangle \langle 00| + y^2 \alpha^2 |00\rangle \langle 00| \\ & + x^2 \beta^2 |01\rangle \langle 01| + x^2 \beta^2 |01\rangle \langle 01| + x^2 \beta^2 |10\rangle \langle 10| + x^2 \beta^2 |10\rangle \langle 01| \\ & + y^2 \beta^2 |00\rangle \langle 00| + x^2 \gamma^2 |00\rangle \langle 00| + x^2 \gamma^2 |00\rangle \langle 00| \\ & + y^2 \gamma^2 |10\rangle \langle 10| + x^2 \beta^2 |01\rangle \langle 10| \} \end{aligned} \quad (11)$$

$$\begin{aligned} \rho_{16} = \frac{1}{N_1} \{ & x^2 \alpha^2 |00\rangle \langle 00| + x^2 \alpha^2 |01\rangle \langle 01| + xy \alpha \gamma |01\rangle \langle 10| \\ & + y^2 \alpha^2 |00\rangle \langle 00| + x^2 \beta^2 |00\rangle \langle 00| + x^2 \beta^2 |00\rangle \langle 00| \\ & + x^2 \gamma^2 |00\rangle \langle 00| + x^2 \gamma^2 |00\rangle \langle 00| + y^2 \gamma^2 |10\rangle \langle 10| + xy \alpha \gamma |10\rangle \langle 01| \} \end{aligned} \quad (12)$$

$$\begin{aligned} \rho_{69} = \frac{1}{N_1} \{ & x^2 \alpha^2 |01\rangle \langle 01| + x^2 \alpha^2 |01\rangle \langle 10| + x^2 \alpha^2 |10\rangle \langle 10| \\ & + x^2 \alpha^2 |10\rangle \langle 01| + y^2 \alpha^2 |00\rangle \langle 00| + x^2 \beta^2 |00\rangle \langle 00| \\ & + x^2 \beta^2 |00\rangle \langle 00| + x^2 \beta^2 |00\rangle \langle 00| + y^2 \beta^2 |00\rangle \langle 00| \\ & + x^2 \gamma^2 |00\rangle \langle 00| + x^2 \gamma^2 |00\rangle \langle 00| + y^2 \gamma^2 |00\rangle \langle 00| \} \end{aligned} \quad (13)$$

$$\begin{aligned} \rho_{86} = \frac{1}{N_1} \{ & x^2 \alpha^2 |00\rangle \langle 00| + x^2 \alpha^2 |01\rangle \langle 01| + x^2 \alpha \beta |01\rangle \langle 10| \\ & + y^2 \alpha^2 |00\rangle \langle 00| + x^2 \beta^2 |10\rangle \langle 10| + x^2 \alpha \beta |10\rangle \langle 01| \\ & + x^2 \beta^2 |00\rangle \langle 00| + y^2 \beta^2 |00\rangle \langle 00| + x^2 \gamma^2 |00\rangle \langle 00| + x^2 \gamma^2 |00\rangle \langle 00| + y^2 \gamma^2 |00\rangle \langle 00| \} \end{aligned} \quad (14)$$

Now, $W_4 < 0$ for $\rho_{15}, \rho_{58}, \rho_{16}, \rho_{69}, \rho_{86}$ for all values of α, β, γ . Since at least one of W_3 and W_4 is negative we can easily infer that the above non local subsystems are entangled.

The local output states are given by,

$$\begin{aligned} \rho_{17} = \rho_{14} = \frac{1}{N_1} \{ & x^2 \alpha^2 |01\rangle\langle 01| + x^2 \alpha^2 |01\rangle\langle 01| + y^2 \alpha^2 |01\rangle\langle 01| \\ & + x^2 \beta^2 |01\rangle\langle 01| + y^2 \beta^2 |01\rangle\langle 01| + x^2 \gamma^2 |01\rangle\langle 01| \\ & + x^2 \gamma^2 |00\rangle\langle 00| + xy \gamma^2 |11\rangle\langle 00| + xy \gamma^2 |10\rangle\langle 01| + y^2 \gamma^2 |11\rangle\langle 11| \} \end{aligned} \quad (15)$$

$$\begin{aligned} \rho_{28} = \rho_{25} = \frac{1}{N_1} \{ & -x^2 \alpha^2 |10\rangle\langle 10| - x^2 \alpha^2 |10\rangle\langle 10| - y^2 \alpha^2 |10\rangle\langle 10| \\ & - x^2 \beta^2 |10\rangle\langle 10| - x^2 \beta^2 |11\rangle\langle 11| + xy \beta^2 |11\rangle\langle 00| + xy \beta^2 |00\rangle\langle 11| \\ & - y^2 \beta^2 |00\rangle\langle 00| - x^2 \gamma^2 |10\rangle\langle 10| - x^2 \gamma^2 |10\rangle\langle 10| - y^2 \gamma^2 |10\rangle\langle 10| \} \end{aligned} \quad (16)$$

$$\begin{aligned} \rho_{36} = \rho_{39} = \frac{1}{N_1} \{ & -x^2 \alpha^2 |10\rangle\langle 10| - x^2 \alpha^2 |11\rangle\langle 11| + xy \alpha^2 |11\rangle\langle 00| \\ & - y^2 \alpha^2 |00\rangle\langle 00| + xy \alpha^2 |00\rangle\langle 11| - x^2 \beta^2 |10\rangle\langle 10| \\ & - x^2 \beta^2 |10\rangle\langle 10| - y^2 \beta^2 |10\rangle\langle 10| - x^2 \gamma^2 |10\rangle\langle 10| - x^2 \gamma^2 |10\rangle\langle 10| - y^2 \gamma^2 |10\rangle\langle 10| \} \end{aligned} \quad (17)$$

Now $W_4 = W_3 = 0$ for ρ_{14}, ρ_{17} and $W_4 > 0, W_3 > 0$ for $\rho_{25}, \rho_{28}, \rho_{36}, \rho_{39}$ clearly indicating the fact that all the local output states are separable.

Though in this work we have considered particular cases and not dealt with all possible measurement results, we have achieved our target of secretly generating five qubit entangled state. One can investigate other measurement results to see how they can be used in other broadcasting problems. This five qubit entangled state will play crucial role as a secret quantum channel between three distant partners.

3 Conclusion:

To summarize, the motivation of our work which was to broadcast five qubit entangled state among three parties secretly. This can be done by applying twice B-H quantum cloning machine followed by subsequent measurement on machine state vectors by three parties. Not only that each of these parties apply local operations after the cloning

transformation to make the local output states separable. Our problem is interesting in the sense that this is the first attempt to broadcast five qubit entangled state secretly. This state can be used by Alice,Bob, Charlie as a five qubit entangled state in various cryptographic protocols viz. quantum key distribution protocols. It is also interesting in the sense that in this protocol we have broadcasted five qubit entangled state in such a way that it is independent of input parameters α, β, γ .

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5 References

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